المجموعات الضبابية المفتوحة $S^M b$ في الفضاءات التوبولوجية الضبابية

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الملخص العربي :

الهدف من هذه الورقة هو تقديم ودراسة الخصائص المختلفة للمجموعات البسيطة والمجموعات البسيطة b المفتوحة في الفضاءات التوبولوجية الضبابية. ونقدم شكلاً جديدًا من الفضاءات الضبابية التامة، تسمى الفضاءات الضبابية البسيطة التامة b والفضاءات الضبابية البسيطة المتراصة b والحصول على بعض خصائصها الأساسية.

FUZZY $S^M b -$ OPEN SETS IN FUZZY TOPOLOGICAL SPACES

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Abstract

The aim of this paper is to introduce and study different properties of simply and simply $b - open$ sets in fuzzy topological spaces. And we introduce a new form of fuzzy compact spaces namely fuzzy simply b −compact spaces, fuzzy simply b-connected and obtain some of their basic properties.

Kay Words

$FS^M -$ open, $FS^M b -$ open, $FS^M b - closed$, $FS^M b -$ Compact & $FS^M b -$ Connected.

Introduction

After Zadeh [13] introduced the concept of a fuzzy subset, Chang [9] used it to define fuzzy topological space. Thereafter, several concepts of general topology have been extended to fuzzy topology and compactness is one such concept. Compactness for fuzzy topological spaces was first introduce by Chang [9]. The concept of b-open sets in fuzzy settings was introduced by S.S. Benchalli and Jenifer [1].The purpose of this paper is to introduce fuzzy b-compact, fuzzy b-closed spaces and fbg-compact spaces, some interesting properties of fuzzy b-closed space are investigated. The notions of fuzzy vector spaces and fuzzy topological vector spaces were introduced in Katsaras and Liu [7]. These ideas were modiÖed by Katsaras [5], and in [6] Katsaras deÖned the fuzzy norm on a vector space. In [8], Krishna and Sarma discussed the generation of a fuzzy vector topology from an ordinary vector topology on vector spaces.

Definition 1.1 [4]

Let X be a non-empty set, a fuzzy set A in X is characterized by a function

$\mu_A : X \to I$, where $I = [0,1]$ which is written as

$A = \{(x, \mu_A(x): x \in X, 0 \le \mu_A(x) \le 1)\}$, the collection of all fuzzy sets in X will be denoted by I^X that is $I^X = \{A: A\ is$ a fuzzy sets in $X\}$ where μ_A is called the membership function.

Definition 1.2 [3]

A fuzzy set x_r in a fuzzy set A is called a fuzzy point if $x(x_0) = r$, if $x = x_q$, and $x(x_0) = 0$, if $x \neq x_q$, $0 < r \leq 1$, such that x and r are the support and the value of the fuzzy point respectively.

Proposition 1-1 [3]

Let B & C are fuzzy subset on A then

1- $B \subseteq C \Leftrightarrow B(x) \leq C(x),\ \forall x \in X.$

2- $B = C \Leftrightarrow B(x) = C(x),\ \forall x \in X.$

3- $B \cap C \Leftrightarrow F(x) = min\{B(x), C(x)\},\ \forall x \in X.$

4- $B = C^C \Leftrightarrow B(x) = A(x) - C(x),\ \forall x \in X.$

Definition 1.3 [10]

A sub set A of a topological space (X, τ), is called simply open if $A = G \cup N$, where G is open set and N is nowhere dense, where $cl\big(int(A)\big) = \emptyset$.

Definition 1.4

For any fuzzy topological space (X, τ), $A \subseteq X$ is called fuzzy dense ($F -$ dense) in X if and only if $cl(A) = X$. The family of all fuzzy dense sets in X will be denoted by $FD(X)$.

Definition 1.5

For any fuzzy topological space (X, τ), $A \subseteq X$ is called fuzzy nowhere dense ($F -$ nowhere) dense in X if $int\big(cl(A)\big) = \emptyset$.

Definition 1.6

A sub set A of a fuzzy topological space (X, τ) is called a fuzzy simply open (FS^MO) if $A = G \cup N$ where G is fuzzy open set and N is fuzzy nowhere dense, where $cl\big(int(A)\big) = \emptyset$.

Definition 1.7 [12]

A fuzzy set A of a fuzzy topological space (X, τ) is said to be fuzzy generalized closed ($Fg -$ closed) if $cl(A) \subseteq O$ whenever $A \subseteq O$ and O is F open in (X, τ). A fuzzy set A of a fuzzy topological space (X,τ) is said to be fuzzy generalized open if its complement A^C is fuzzy generalized closed.

Definition 1.8 [**11**]

A fuzzy set A in a fuzzy topological space X is said to be fuzzy $b-$ open set if and only if $A \leq \left(int\left(cl(A)\right)\right) \cup \left(cl\left(int(A)\right)\right)$.

Definition 1.9

A subset A of a topological space (X,τ), is called fuzzy simply $b-$ open (FS^Mb- open) if $A = G \cup N$, where G is fuzzy $b-$ open set and N is fuzzy nowhere dense, where $cl\left(int(A)\right) = \emptyset$.

Remark 1.1

We denote the class of all fuzzy simply $b-$ open set by $FS^MbO(X)$. The complement of fuzzy simply $b-$ open sets are called FS^Mb- closed sets, which denoted by $FS^MbC(X)$.

Example 1.1

Let $X = \{a,b,c\}, (\tau,X)$ be fuzzy topological space on X, s. t. $\tau = \{0,1,A,B\}$,
$A = \{(a,.7),(b,.3),(c,.1)\}$, $B = \{(a,.7),(b,0),(c,0)\}$, $D = \{(a,.8),(b,.7),(c,.7)\}$, so D is fuzzy $b-$ open set in FTS (τ,X), but not fuzzy S^Mb- open set.

Definition 1.10[**1**]

Let A be a fuzzy set in a fuzzy topological space (X,τ). Then a fuzzy $b-$ interior and fuzzy $b-$ closure of A is denoted by $bint(A)$ and $bcl(A)$ defined by

$bint(A) = \bigcup\{G: G \subseteq A, G\ is\ Fb-$ open set in $X\}$ &

$bcl(A) = \bigcap\{F: A \subseteq F, F\ is\ Fb-$ closed set in $X\}$.

Definition 1.11

Let A be a fuzzy set in a fuzzy topological space (X, τ). Then fuzzy simply $b -$ interior and fuzzy simply $b -$ closure of A is denoted by $S^M\ bint(A)$ and $S^M\ bcl(A)$ defined by
$S^M bint(A) = \bigcup\{G: G \subseteq A, G\ is\ FS^M b -$ open set in $X\}$ &
$S^M bcl(A) = \bigcap\{F: A \subseteq F, F\ is\ S^M\ Fb -$ closed set in $X\}$**.**

Definition 1.12 [**2**]

A fuzzy set A in a fuzzy topological space X is called fuzzy generalized $b - closed$ (briefly$Fgb -$ closed) fuzzy set if $bcl(A) \leq B$ when ever $A \leq B$ & B is fuzzy $b -$ open in (X, τ).

Definition 1.13

Let A be a fuzzy set in a fuzzy topological space (X, τ). Then A is called a fuzzy simply $b -$ closed set in X, if $S^M\ bcl(A) \subseteq D$ when ever $A \subseteq D$ and D is fuzzy generalized open set in X.

Example 1.2

Let $X = \{a, b, c\}, (\tau, X)$ be fuzzy topological space on$X$, s. t. $\tau = \{0,1, A, B, C\}$,
$A = \{(a, 1), (b, 0), (c, 0)\}$, $B = \{(a, 0), (b, 1), (c, 0)\}$, $C = \{(a, 1), (b, 1), (c, 0)\}$, $E = \{(a, .2), (b, .7), (c, .8)\}$, so E is $FS^M bO(X)$ s.t. $E = G \cup N$, where
$G = \{(a, .2), (b, .7), (c, .6)\}$, $N = \{(a, 0), (b, 0), (c, .8)\}$, but it is not $FS^M O(X)$ since $G \notin FO(X)$.

Definition 1.14

Let (X, τ) be fuzzy topological space, a family W of fuzzy sets is $S^M -$ open
cover of a fuzzy sets A if and only if $A \subseteq \bigcup\{G: G \in W\}$ and each member of W is an $S^M -$ open fuzzy set. A sub cover of W is a sub family which is also cover.

Definition 1.15

Let (X,τ) be fuzzy topological space, a family W of fuzzy sets is $S^{M}b$ −open cover of a fuzzy sets A if and only if $A \subseteq \cup\{G: G \in W\}$ and each member of W is a $S^{M}b$ − open fuzzy set. A sub cover of W is a sub family which is also cover.

Remark 1.2

Every fuzzy open cover is a fuzzy simply open ($FS^{M}b$ −open) cover. But the converse is not true in general.

Example 1.3

Let $X = \{a, b\}$ and $B_1, B_2, .., B_{11}$ are fuzzy sets of A such that, $\tau = \{\emptyset, A, B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8, B_9\}$, $A = \{(a, .8), (b, .9)\}$,
$B_1 = \{(a, .8), (b, 0)\}$, $B_2 = \{(a, 0), (b, .7)\}$,
$B_3 = \{(a, .8), (b, .7)\}$,
$B_4 = \{(a, .1), (b, .9)\}$, $B_5 = \{(a. 6,), (b, 0)\}$, $B_6 = \{(a, .1), (b, 0)\}$,
$B_7 = \{(a, .6), (b, .9)\}$, $B_8 = \{(a, .1), (b, .7)\}$,
$B_9 = \{(a, .6), (b, .7)\}$,
$B_{10} = \{(a, 0), (b, .8)\}, B_{11} = \{(a, .7), (b, 0)\}$, then
$FbO(X) = \{ B_4\}$ & $FbS^{M}O(X) = \{A, B_4, B_7\}$, note that B_7 is fuzzy simply b − open set but its not fuzzy b − open set.

2. Fuzzy Simply (b - Simply) Closed Sets

Definition 2.1

Let A be a fuzzy set in a fuzzy topological space (X, τ), Then A is called a Fb − closed set in X if $bcl(A) \subseteq O$, whenever $A \subseteq O$ and O is fuzzy generalized open set in X.

Definition 2.2

Let A be a fuzzy set in a fuzzy topological space (X, τ), Then A is called a fuzzy simply b − closed ($FS^{M}b$ − closed) set in X if $S^M bcl(A) \subseteq G$, whenever $A \subseteq G$ and G is fuzzy generalized open set in X.

Theorem 2.1

Every fuzzy simply $b-$ closed ($FS^M b-$ closed) set in a fuzzy topological space
(X,τ) is fuzzy simply $b-$ closed ($FS^M b-$ closed) set.

Proof

Let A be a fuzzy simply $b-$ closed set in a fuzzy topological space (τ, X). Suppose that $A \subseteq D$ and D is fuzzy generalized open set in X, since A is a $FS^M b-$ closed set, hence $S^M bcl(A) = A$. Thus $S^M bcl(A) = A \subseteq D$, and hence A is $FS^M b-$ closed set. But the converse may not true in general.

Lemma 2.1

Let A be a fuzzy set in a fuzzy topological space (X,τ). Then $A \cup int\left(cl\big(int(A)\big)\right) \subseteq bcl(A)$.

Theorem 2.2

Let A be a fuzzy set in a fuzzy topological space (X,τ). If A is fuzzy generalized open and fuzzy simply $b-$ closed, then A is fuzzy simply $b-$ closed.

Proof

Since A is fuzzy generalized open and fuzzy simply $b-$ closed, it follows that
$A \cup int\left(cl\big(int(A)\big)\right) \subseteq bcl(A) \subseteq A$. Hence $int\left(cl\big(int(A)\big)\right) \subseteq A$ and A is fuzzy simply $b-$ closed.

Lemma 2.2

Let A be an fuzzy set in an fuzzy topological space (X,τ). Then $S^M bcl\big(S^M bcl(A)\big) = S^M bcl(A)$.

Theorem 2.3

Let A be a fuzzy simply $b-$ closed set in a F topological space (X,τ). If B is a fuzzy set in X such that $A \subseteq B \subseteq bS^M c(A)$, then B is also fuzzy simply $b-$ closed.

Proof

Let B be a fuzzy set in a fuzzy topological space (X,τ) such that $B \subseteq G$ and G is a fuzzy generalized open set in X. So $A \subseteq G$. Since A is a $FS^M b-$ closed, Then by lemma (2.2) $S^M bcl(A) \subseteq S^M bcl\big(S^M bcl(A)\big) = S^M bcl(A) \subseteq G$. Therefore B is a fuzzy simply $b-$ closed in X.

Definition 2.3

A fuzzy set A in a fuzzy topological space (X,τ) is called a fuzzy simply $b-$ open if and only if its complement A^C is fuzzy simply $b-$ closed.

Theorem 2.4

A fuzzy set A in a fuzzy topological space (X,τ) is called a fuzzy simply $b-$
open if $G \subseteq S^M bint(A)$ whenever $G \subseteq A$ and G is fuzzy generalized closed set in X.

Proof

Let a fuzzy set A in a fuzzy topological space (X,τ) be a fuzzy simply $b-$ open and G is a fuzzy $g-$ closed set in X such that $G \subseteq A$. Then $A^C \subseteq G$, where A^C is a fuzzy simply $b-$ closed and G^C is a fuzzy $g-$ open in X. Therefore by definition (2.1) we have $S^M bcl(A^C) \subseteq G^C$. Then $(G^C)^C \subseteq \big(S^M bcl(A^C)\big)^C$.
i.e. $G \subseteq S^M bint(A^C)^C = S^M bint(A)$.

Theorem 2.5

Let A be a fuzzy $b-$ closed set in a fuzzy topological space (X,τ). If B is a fuzzy set in X such that $bint(A) \subseteq B \subseteq A$, then B is also fuzzy $b-$ open.

Proof

Let B be a fuzzy set in a fuzzy topological space (X,τ), such that $G \subseteq B$ and G is fuzzy $g-$ closed set in X. Then $G \subseteq A$. Since A is fuzzy $b-$ closed, hence $G \subseteq bint(A)$. So $G \subseteq bint(A) \subseteq B$. Therefore B is fuzzy $b-$ open in X.

3. Some Types of Near Normality Based on $FS^M b$ −Open Sets

In this section we introduce some types of normality in fuzzy topological space (X,τ) based on $FS^M\ O(X)$, $FS^M\ bO(X)$.

Definition 3.1

A fuzzy topological space (X,τ) is called a fuzzy b −normal (Fb −normal) if $\forall\ F_1, F_2 \in FbC(X)$, $F_1\overline{q}F_2, \exists\ U,V \in FbO(X), U\overline{q}V$ such that $\mu_{F_1}(x) < \mu_U(x)$ & $\mu_{F_2}(x) < \mu_V(x)$.

Example 3.1

From the example *(1.2)* let $D = \{(a,.2),(b,.7),(c,.6)\}$, so D is fuzzy $b-$ open set in fuzzy topological space, and τ is fuzzy b-normal space.

Definition 3.2

A fuzzy topological space (X,τ) is called fuzzy simply normal (FS^M −normal) if $\forall\ F_1, F_2 \in FS^M C(X)$, $F_1\overline{q}F_2, \exists\ U,V \in FS^M O(X), U\overline{q}V$ such that
$\mu_{F_1}(x) < \mu_U(x)$ & $\mu_{F_2}(x) < \mu_V(x)$.

Definition 3.3

A fuzzy topological space (X,τ) is called fuzzy simply$b-$ normal ($FS^M b$ −normal) if $\forall\ F_1, F_2 \in FbC(X)$, $F_1\overline{q}F_2, \exists\ U,V \in FS^M bO(X), U\overline{q}V$ such that

$\mu_{F_1}(x) < \mu_U(x)$ & $\mu_{F_2}(x) < \mu_V(x)$.

Definition 3.3

Let (X,τ) be any fuzzy topological space and $A \subseteq X$, we define the simply boundary ($S^M BN$) of A as follows
$S^M BN(A) = S^M cl(A) \cap S^M cl(X - A)$.

Theorem 3.1

For a fuzzy topological space (X,τ) and $A, B \subseteq X$. Then the following statements hold

1- $S^M BN(A) = S^M BN(X - A)$.
2- $S^M BN(A) = S^M cl(A) - S^M int(A)$.
3- $S^M BN(A) \cap S^M int(A) = \emptyset$.
4- $S^M BN(A) \cup S^M int(A) = S^M cl(A)$.

Proof

1 are obvious

2- Since $S^M cl(X - A) = X - \cap$, then $S^M BN(A) = S^M cl(A) \cap S^M cl(X - A)$

$$= S^M cl(A) \cap [X - S^M int(A)]$$
$$= S^M cl(A) - [S^M cl(A) \cap S^M int(A)] = S^M cl(A) - S^M int(A).$$

3, 4 obvious from (2).

Definition 3.4

For any topological space (X,τ), a subset A of X is said to be fuzzy simply nowhere dense (FS^M −nowhere dense) if $S^M int\left(S^M cl(A)\right) = \emptyset$.

Theorem 3.2

For any fuzzy topological space (X,τ) and $A \subseteq X$, the following statement hold

1- $A \in S^M O(A) \Leftrightarrow A \cap S^M BN(A) = \emptyset$.
2- $A \in S^M C(A) \Leftrightarrow S^M BN(A) \subset A$.

3- $A \in S^M O(A) \cap S^M C(A) \Leftrightarrow S^M BN(A) = \emptyset$.

Proof

1- Let $A \in FS^M O(X)$, i.e. $A = FS^M int(A)$. Then
$A \cap FS^M BN(A) = FS^M int(A) \cap FS^M BN = \emptyset$.
Conversely, let $A \cap FS^M BN(A) = \emptyset$, then $\emptyset = FS^M int(A) = A - FS^M int(A)$, so $A = FS^M int(A)$. Then $A \in FS^M O(X)$.

2- Let $A \in FS^M C(X)$, i.e. $A =$. Then $FS^M BN(A) = FS^M cl(A)$
$FS^M BN(A) = FS^M cl(A) \cap Fcl(X - A) = A \cap FS^M cl(A) \subset A$
Conversely, let $FS^M BN(A) \subset A$. Since
$FS^M int(A) = FS^M BN(A) \cup FS^M int(A) \subset A \cup FS^M int(A) - A$. Therefore
$A = FS^M cl(A)$. So $A \in FS^M C(X)$.

3- Let $A \in FS^M O(X) \cap FS^M C(X)$, i.e. $A \in FS^M O(X)$ and $A \in FS^M C(X)$, from (1 , 2) we have $A \cap FS^M BN(A) = \emptyset$, and $FS^M BN(A) \subset A$.Consequently, $FS^M BN(A) = \emptyset$. Conversely, let $FS^M BN(A) = \emptyset$. Since $FS^M BN(A) = \emptyset \subset A$, (by2) we have $A \in FS^M C(X)$. Also $A \cap FS^M BN(A) = \emptyset$,(by 1) we have $A \in FS^M O(X)$. Therefore $A \in FS^M C(X) \cap FS^M O(X)$.

4. $FS^M b -$ *Compact and* $FS^M b -$ *Connected*

Definition 4.1

A fuzzy topological space (X, τ) is called to be fuzzy simply compact ($FS^M -$ compact) $\Leftrightarrow$ every $FS^M -$open cover of X has a finite sub cover.

Definition 4.2

A fuzzy topological space (X, τ) is called to be fuzzy simply $b -$ compact ($FS^M b -$ compact) $\Leftrightarrow$ every $FS^M b -$open cover of X has a finite sub cover.

Definition 4.3

A fuzzy sub set A of a fuzzy topological space (X, τ) is called to be fuzzy simply compact ($F\ S^M -$ compact) set relative to X if every $FS^M -$ open cover of A has a finite sub cover.

Theorem 4.1

A $FS^M b -$ closed subset of an $FS^M b -$ compact space (X, τ) is $FS^M b -$ compact.

Proof

Let A be a $FS^M b -$ closed set in X. Then $X - A$ a $FS^M b -$ open set. Let
$\{G_i : i \in I\} \subset FS^M bO(X)$. Since X is $FS^M b -$compact, then there exist a finite fuzzy subset $I_\propto$ of I, such that $X = (X - A) \cup \left(\bigcup_{i \in I_\propto} G_i\right) \in FS^M bO(X)$. Hence
$\mu_A(x) < \mu_{\bigcup_{i \in I_\propto} G_i}(x)$, and A be a $FS^M b -$ compact subset of X.

Theorem 4.2

Let A & B be fuzzy subset of a fuzzy topological space (X, τ). If A is $FS^M b -$
compact relative to X and B is $FS^M b -$ closed set in (X, τ), then $A \cap B$ is $FS^M b -$ compact relative to X.

Proof

Let $\{U_i : i \in I\}$ be a $FS^M b -$ open cover of $A \cap B$. Since $B \in FS^M bC(X)$, then $X - B \in FS^M bO(X)$. Therefore $(X - B) \cup (\{U_i : i \in I\}) \subset FS^M bO(X)$, which is a cover of A. Since A is $FS^M b -$ compact relative to X, then there exist a finite fuzzy subset I_α of I such that $\mu_A(x) < \mu_{\bigcup \{U_i : i \in I_\alpha\} \cup (X-B)}(x)$. Then $\mu_{A \cap B}(x) < \mu_{\bigcup \{U_i : i \in I_\alpha\} \cup (X-B) \cap B}(x) < \mu_{\bigcup \{U_i : i \in I_\alpha\} \cup \emptyset}(x) <$
$\mu_{\bigcup \{U_i : i \in I_\alpha\}}(x)$. Hence $A \cap B$ is $FS^M b -$ compact relative to X.

Theorem 4.3

Every $FS^M b-$ open subset of X is $FS^M b-$ compact $\Leftrightarrow$ it is $FS^M b-$ compact relative to X.

Proof

Let U be $FS^M b-$ compact and let $\{U_i: i \in I\}$ be $FS^M b-$ open cover of U. Since U is $FS^M b-$ compact then $U \subset \bigcup\{U_i: i \in I\}$, and then there exist a finite fuzzy subset I_α of I such that $U \subset \bigcup\{U_i: i \in I_\alpha\}$. Then U is $FS^M b-$ compact relative to X. Conversely, let $\{U_i: i \in I\}$ be $FS^M b-$ open cover of U since U, is $FS^M b-$ compact set relative to X. Thus $\mu_U(x) < \mu_{\bigcup\{U_i: i\in I\}}(x)$ and hence there exist a finite fuzzy subset $I_\alpha$of I such that $\mu_U(x) < \mu_{\bigcup\{U_i: i\in I_\alpha\}}(x)$. This indicates that U is $FS^M b-$ compact.

Definition 4.4

Two non empty fuzzy sub set A & B in a fuzzy topological space (X,τ) are said to be fuzzy simply$b-$ separated ($FS^M b-$ separated) if $A\overline{q}S^M cl(B)$ & $B\overline{q}S^M cl(A)$.

Remark 4.1

Any two $FS^M b-$ separated sets are always disjoint but the converse need not true in general.

Remark 4.2

Each two separated sets are $FS^M b-$ separated, since $FS^M bcl(A) \subset bcl(A)$.

Theorem 4.4

Let A & B be non empty fuzzy subset of a fuzzy topological space (X,τ). Then

the following statement holds

1- If A & B are $FS^M b-$ separated and A_1 , B_1 are non empty fuzzy sets such that $\mu_{A_1}(x) < \mu_A(x)$ and $\mu_{B_1}(x) < \mu_B(x)$ then A_1 & B_1 are also $FS^M b -$ separated.

2- If $A\overline{q}B$ and either both of A & $B \in FS^M bO(X)$ or A & $B \in FS^M bC(X)$, then

A & B are $FS^M b -$ separated.

3- If either both of A & $B \in FS^M bO(X)$ or A & $B \in FS^M bC(X)$, and if

$H = A \cap (X - B)$ and $G = B \cap (X - A)$ then H& G are $FS^M b -$ separated.

Proof

1- Since $\mu_{A_1}(x) < \mu_A(x)$, so $\mu_{S^M bcl(A_1)}(x) < \mu_{S^M bcl(A)}(x)$ then $B_1\overline{q}S^M bcl(A_1)$

$B_1\overline{q}S^M bcl(A)$, $B\overline{q}S^M bcl(A)$. Hence $B_1\overline{q}S^M bcl(A_1)$. Similarity, since $\mu_{B_1}(x) < \mu_B(x)$, then $\mu_{S^M bcl(B_1)}(x) < \mu_{S^M bcl(B)}(x)$. Consequently, $A_1\overline{q}S^M cl(B_1)$, $A_1\overline{q}S^M cl(B)$, $A\overline{q}S^M bcl(B)$. Then $A_1\overline{q}S^M cl(B)$. Therefore A & B are $FS^M b -$ separated.

2- Let $A, B \in FS^M bO(X)$. Thus $X - A, X - B \in FS^M bC(X)$. Since $A\overline{q}B$, then

$\mu_A(x) < \mu_{X-B}(x)$. Therefore$\mu_{S^M bcl(A)}(x) < \mu_{X-B}(x) = X - B$, since

$X - B \in S^M bcl(A)$. Then $B\overline{q}S^M bcl(A) \subset B\overline{q}(X - B)$, then $B\overline{q}S^M bcl(A)$. Similarly $A\overline{q}S^M bcl(B)$. Hence A & B are $FS^M b -$ separated.

3- Let $A, B \in FS^M bO(X)$. Thus $X - A, X - B \in FS^M bC(X)$. Since $\mu_A(x) < X - B$, and $\mu_{S^M bcl(H)}(x) < \mu_{S^M bcl(X-B)}(x) = X - B$, and $B\overline{q}S^M bcl(H)$, $B\overline{q}(X - B)\overline{q}\ S^M bcl(H)$, $G\overline{q}S^M bcl(H)$. Since $G\overline{q}X - A$. Then $\mu_{S^M bcl(G)}(x) < \mu_{S^M bcl(X-A)}(x) = X - A$,

$H\overline{q}S^M bcl(G) \subset H\overline{q}(X-A), A\overline{q}S^M bcl(G)$. Therefore $(X-B)\overline{q}A\overline{q}\ A \cap (X-B) \cap S^M bcl(B) \cap S^M bcl(X-A)H\overline{q}S^M bcl(G)$. Thus $H \& G$ are $FS^M -$ separated. Let $A, B \in FS^M bC(X)$. Thus $A = S^M bcl(A)$, $B = S^M bcl(B)$. Since $\mu_H(x) < \mu_{(X-B)}(x)$, then $H\overline{q}S^M bcl(B)$. Since$\mu_{S^M bcl(G)\cap H}(x) < \mu_{A\cap(X-B)\cap S^M bcl(B)\cap S^M bcl(X-A)}(x)$, s.t.
$A \cap (X-B)\overline{q}S^M bcl(B)\overline{q}S^M bcl(X-A) = A\overline{q}(X-B)\overline{q}S^M bcl(X-B)$. Therefore $S^M bcl(G)\overline{q}H$. Similarly $S^M bcl(H)\overline{q}G$. Hence $H \& G$ are $FS^M b -$ separated sets.

Definition 4.5

A sub set G of a fuzzy topological space (X,τ) is said to be fuzzy simply $b-$ ($FS^M b -$ connected) relative to (X,τ). If there are no $FS^M b -$ open sub set A & B of X such that A & B are $FS^M b -$ separated relative to (X,τ) and $G = A \cup B$.

Remark 4.3

For any fuzzy topological space (X,τ) every $FS^M b -$ connected is connected. But the converse is not true in general.

Theorem 4.5

Two none empty fuzzy sets A & B in an fuzzy topological space (X,τ) are $FS^M b -$ separated if and only if there exist $U, V \in FS^M bO(X)$ such that
$\mu_A(x) < \mu_U(x), \mu_B(x) < \mu_V(x), A\overline{q}V$ & $B\overline{q}U$.

Proof

Let A and B be $FS^M b -$ separated sets. Let $V = X - S^M bcl(A)$ and
$U = X - S^M bcl(B)$. Then $U, V \in S^M bO(X)$, such that $\mu_A(x) < \mu_U(x)$,
$\mu_B(x) < \mu_V(x)$ & $A\overline{q}V, B\overline{q}U$. Conversely, let, $U, V \in S^M bO(X), \mu_A(x) < \mu_U(x)$,

$\mu_B(x) < \mu_V(x)$ & $A\overline{q}V$, $B\overline{q}U$. Since $X - V, X - U \in S^M bC(X)$, then
$\mu_{S^M bcl(A)}(x) < \mu_{X-V}(x) < \mu_{X-B}(x)$& $\mu_{S^M bcl(B)}(x) < \mu_{X-U}(x) < \mu_{X-A}(x)$, therefore $S^M bcl(A)\overline{q}B$&$S^M bcl(B)\overline{q}A$.
Hence A & B are $FS^M b -$ separated sets.

Theorem 4.6

Every $Fb -$ compact space is $F -$ compact space.

Proof

Suppose that (X, τ) is a fuzzy$b -$ compact space, and let the collection
$W = \{A_i : A_i \in I^x, i \in I\}$ be a fuzzy open cover of X. Since $Sup\{\mu_A(x)\} = 1$, then $X = \cup A$. By (every fuzzy open cover is a fuzzy $b -$ cover) then W is fuzzy $b -$ open cover of a fuzzy $b -$ compact space, then X has a finite sub cover which belong to W Therefore X is a fuzzy compact space.

Theorem 4.7

Every $Fb -$ closed sub set of a $Fb -$ compact space is $F -$ compact.

Proof

Suppose that (X, τ) is a fuzzy $b -$ compact space, and G is a fuzzy compact. Let
$W = \{A_i : A_i \in I^x, i \in I\}$ be a fuzzy open cover of G in(X, τ).
Since G is a subset of W then $\mu_G(x) \leq sup\{\mu_A(x)\}$. So that, W is a fuzzy $b -$ open cover of G, since G is a fuzzy $b -$ closed subset of X, then G^C is a fuzzy $b -$ open subset of X. Therefore $W \cup G^C$ is a fuzzy open cover of X, which is fuzzy $b -$ compact space. Then there exist members of W such that $X = A_i \cup \{G^C\}$, i.e. X has two finite sub cover $\{A_i, G^C\}$ since $\mu_G(x) \leq 1$,
then $G \subseteq X$ and G^C cover no part of G. Hence,

$\mu_G(x) \leq sup\{\mu_A(x)\}$, so $G \subseteq A_i$. Therefore G is a fuzzy compact.

Remark 4.4

A $Fb-$ closed sub set of a $F-$ compact need not be compact.

Proposition 4.1

Let (X,τ) be an $F-$ topological space if A and B are two $Fb-$ compact sub set of X, then $A \cup B$ is also $Fb-$ compact.

Proof

Let $W = \{A_i : A_i \in I^x, i \in I\}$ be a $Fb-$ open cover of $A \cup B$, then
$\mu_{A\cup B}(x) \leq Sup\{\mu_{A_i}(x) : i \in I\}$, so $Max\{\mu_A(x), \mu_B(x)\} \leq Sup\{\mu_{A_i}(x) : i \in I\}$, therefore $A \cup B \subseteq A_i, i \in I$. Since $\mu_A(x) \leq \mu_{A\cup B}(x)$, then $A \subseteq A \cup B$, also
$\mu_B(x) \leq \mu_{A\cup B}(x)$ i.e. $B \subseteq A \cup B$. So $\{A_i, i \in I\}$ is a fuzzy $b-$ open cover of A, and a fuzzy $b-$ open cover of B. Since A & B are two fuzzy $b-$ compact sets, then there exist a finite sub cover $(A_1, A_2, \dots, A_n)$, which covering A belong to $\{A_i, i \in I\}$. Then$\mu_A(x) \leq Max\{\mu_{A_i}(x) : i \in I\}$, so $A \subseteq A_i, i = 1,2,\dots,n$, and there exist a finite sub cover $(A_1, A_2, \dots, A_m)$ which covering B belong to $\{A_i, i \in I\}$. Then
$\mu_B(x) \leq Max\{\mu_{A_i}(x) : i \in I\}$, so $B \subseteq A_i : i = 1,2,\dots,m$. Therefore
$\mu_{A\cup B}(x) \leq Sup\{\mu_{A_i}(x) : i = 1,2,\dots,n+m\}$, then $A \cup B \subseteq A_i, i = 1,2,\dots,n+m$. Then $A \cup B$ is $Fb-$ compact.

Proposition 4.2

Let (X,τ) be an $F-$ topological space if A and B are two $Fb-$ compact sub set
of X, then $A \cap B$ need not be Fb compact.

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